The Quantum Computing CheatSheet

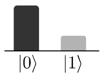
Qubit

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$

$\alpha, \beta \in \mathbb{C}$

Superposition

$|\psi\rangle$ is $|\alpha|^2$% 0 and $|\beta|^2$% 1



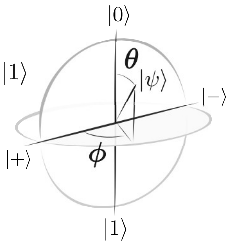
Bloch Sphere

$|\psi\rangle = cos(\frac{\theta}{2})|0\rangle + e^{i\phi}sin(\frac{\theta}{2})|1\rangle$

$\theta, \phi \in \mathbb{R}$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$



QRegister

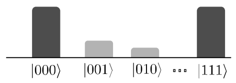
Several qubits forms a Register ⚠

$|\Psi\rangle = |\psi_2\psi_1\psi_0\rangle = |\psi_2\rangle \otimes |\psi_1\rangle \otimes |\psi_0\rangle$

n qubits represent 2^n states

$$|\Psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle = \begin{bmatrix}a \\ b \\ c \\ d \\ e \\ f \\ g \\ h\end{bmatrix}$$

Superposition of all states



Entanglement

The value of a qubit depends on the value of another
=> System state cannot be represented as a product of qubits states
=> System can be pure, but individual qubits are in a mixed state.

$$\underbrace{\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle}_{\text{Pure state}} \neq \underbrace{(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)}_{\text{Mixed state}}$$

Matrix Product $\times$

$$\begin{bmatrix}1 & 2 \\ 3 & 4\end{bmatrix} \times \begin{bmatrix}0 & 5 \\ 6 & 7\end{bmatrix} = \begin{bmatrix}1\times 0 + 2\times 6 & 1\times 5 + 2\times 7 \\ 3\times 0 + 4\times 6 & 3\times 5 + 4\times 7\end{bmatrix}$$

Linear:

$\lambda(A \times B) = (\lambda A) \times B = A \times (\lambda B)$

$(A + B) \times C = (A \times C) + (B \times C)$

Non commutatif:

$A \times B \neq B \times A$

Tensor Product $\otimes$

$$\begin{bmatrix}1 & 2 \\ 3 & 4\end{bmatrix} \otimes \begin{bmatrix}0 & 5 \\ 6 & 7\end{bmatrix} = \begin{bmatrix}1\begin{bmatrix}0 & 5 \\ 6 & 7\end{bmatrix} & 2\begin{bmatrix}0 & 5 \\ 6 & 7\end{bmatrix} \\ 3\begin{bmatrix}0 & 5 \\ 6 & 7\end{bmatrix} & 4\begin{bmatrix}0 & 5 \\ 6 & 7\end{bmatrix}\end{bmatrix}$$

Linear:

$\lambda(A \otimes B) = (\lambda A) \otimes B = A \otimes (\lambda B)$

$(A + B) \otimes C = (A \otimes C) + (B \otimes C)$

Non commutatif:

$A \otimes B \neq B \otimes A$

Density Matrix

$$\rho = |\Psi\rangle\langle\Psi| = \begin{bmatrix}a \\ b \\ c \\ d\end{bmatrix} \times \begin{bmatrix}a^* b^* c^* d^*\end{bmatrix} = \begin{array}{c c} & \begin{array}{cccc}|00\rangle & |01\rangle & |10\rangle & |11\rangle\end{array} \\ \begin{array}{c}|00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle\end{array} & \begin{bmatrix}|a|^2 & ab^* & ac^* & ad^* \\ ba^* & |b|^2 & bc^* & bd^* \\ ca^* & cb* & |c|^2 & cd^* \\ da^* & db^* & dc^* & |d|^2\end{bmatrix}\end{array}$$

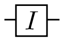
Entanglement level of $|00\rangle$ relative to $|01\rangle$, $|10\rangle$ and $|11\rangle$

Probabilities
$Tr(\rho) = 1$

QGates

Identity

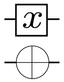
$|0\rangle \Rightarrow |0\rangle$
$|1\rangle \Rightarrow |1\rangle$



$$\begin{array}{cc} \begin{array}{cc}|0\rangle & |1\rangle\end{array} & \\ \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} & \begin{array}{c}|0\rangle \\ |1\rangle\end{array}\end{array}$$

PauliX

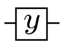
$|0\rangle \Rightarrow |1\rangle$
$|1\rangle \Rightarrow |0\rangle$



$$\begin{array}{cc} \begin{array}{cc}|0\rangle & |1\rangle\end{array} & \\ \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix} & \begin{array}{c}|0\rangle \\ |1\rangle\end{array}\end{array}$$

PauliY

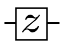
$|0\rangle \Rightarrow i|1\rangle$
$|1\rangle \Rightarrow -i|0\rangle$



$$\begin{array}{cc} \begin{array}{cc}|0\rangle & |1\rangle\end{array} & \\ \begin{bmatrix}0 & -i \\ i & 0\end{bmatrix} & \begin{array}{c}|0\rangle \\ |1\rangle\end{array}\end{array}$$

PauliZ

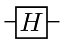
$|0\rangle \Rightarrow |0\rangle$
$|1\rangle \Rightarrow -|1\rangle$



$$\begin{array}{cc} \begin{array}{cc}|0\rangle & |1\rangle\end{array} & \\ \begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix} & \begin{array}{c}|0\rangle \\ |1\rangle\end{array}\end{array}$$

Hadamard

$|0\rangle \Rightarrow |+\rangle \Rightarrow |0\rangle$
$|1\rangle \Rightarrow |-\rangle \Rightarrow |1\rangle$

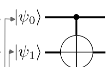


$$\frac{1}{\sqrt{2}} \begin{array}{cc} \begin{array}{cc}|0\rangle & |1\rangle\end{array} & \\ \begin{bmatrix}1 & 1 \\ 1 & -1\end{bmatrix} & \begin{array}{c}|0\rangle \\ |1\rangle\end{array}\end{array}$$

CNOT ⚠

$|00\rangle \Rightarrow |00\rangle$
$|01\rangle \Rightarrow |11\rangle$
$|10\rangle \Rightarrow |10\rangle$
$|11\rangle \Rightarrow |01\rangle$

Control
Target

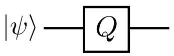


$$\begin{array}{cc} \begin{array}{cccc}|00\rangle & |01\rangle & |10\rangle & |11\rangle\end{array} & \\ \begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0\end{bmatrix} & \begin{array}{c}|00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle\end{array}\end{array}$$

Apply QGate to qubits

Single qubit

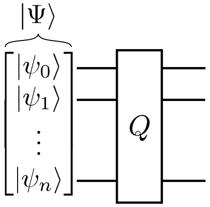
Multiply qubit complex coordinates by gate matrix



$Q(|\psi\rangle) = M \times |\psi\rangle = \begin{bmatrix}a & b \\ c & d\end{bmatrix} \times \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$

Multiple qubits

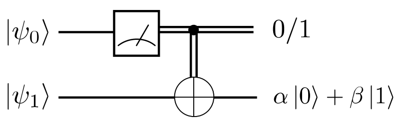
Same than single qubit, except that matrix is $2^n * 2^n$
The circuit act on the whole state, not on each individual qubits.



$$Q\left(\overbrace{\begin{bmatrix}|\psi_0\rangle \\ |\psi_1\rangle \\ \vdots \\ |\psi_n\rangle\end{bmatrix}}^{|\Psi\rangle}\right) = \overset{2^n * 2^n}{M} \times \begin{bmatrix}|\psi_0\rangle \\ |\psi_1\rangle \\ \vdots \\ |\psi_n\rangle\end{bmatrix}$$

Measuring qubits

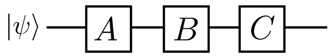
After measurement, qubits are no more in superposition. They collapse to classical bits.
Single lines are qubits. Doubled lines are classical bits.



QCircuits

Single qubit

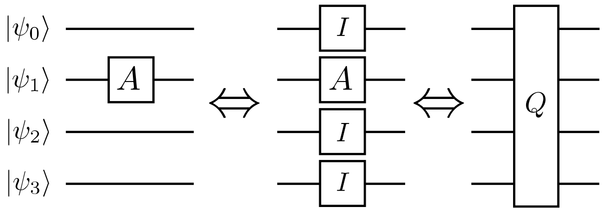
To compose QGates, we multiply matrices.



$Q(|\psi\rangle) = M \times |\psi\rangle = C \times B \times A \times |\psi\rangle$

Multiple qubits

To build a matrix equivalent to the whole circuit, we use the tensor product of each single qubit's QGates.⚠



$$Q(|\Psi\rangle) = \underbrace{(I \otimes I \otimes \overset{2^1 * 2^1}{A} \otimes I)}_{2^4 * 2^4} \times |\Psi\rangle$$

⚠ Computer science convention used: 1st qubit rightmost.
Mathematicians generally use the 1st qubit leftmost.